## To the Editor:

Referring to the article "Two-Phase Flow in Porous Media: Property Identification and Model Validation" Kulkarni et al., (Nov. 1998), we would like to bring attention to the following information.

Pressure formulation is used for description of two-phase flow in porous core plugs. Although it is valid, this approach leads to solving two separate partial differential equations, subject to a pair of initial conditions, a pair of core injection face boundary conditions, and three different pairs of core production face boundary conditions.

However, for the core and fluid conditions considered in their study, an alternative, practical, and simplified formulation, as described below, could be used. This results in a single partial differential equation, subject to single initial, as well as injection and production boundary conditions, which can be solved with a much less computational effort.

Similar to Kulkarni et al. (1998), consider that the core, water, and oil properties remain constant; the core is initially saturated with water, the flow begins by injecting oil at a constant rate to displace the water, the pressure at the production outlet face of the core is constant, and the flow is one-dimensional and horizontal. Under these conditions, the immiscible displacement in a core plug can be described by the following equations (Richardson, 1961; Dullien, 1992; Civan, 1996)

$$\phi\frac{\partial S_w}{\partial t}+\frac{dF_w}{dS_w}u\frac{\partial S_w}{\partial x}+\frac{\partial}{\partial x}\times\left[F_w\frac{k_{ro}}{\mu_o}\frac{dP_c}{dS_w}K\frac{\partial S_w}{\partial x}\right]=0,\quad 0\le x\le L,\ t>0 \quad (1)$$

subject to the initial condition given by

$$S_w=S_w^*(x),\quad 0\le x\le L,\ t=0 \quad (2)$$

and the injection and production face boundary conditions given, respectively, by

$$\frac{\partial S_w}{\partial x}=-\frac{\mu_o(u_o)_{inj}}{Kk_{ro}}\left[\frac{dP_c}{dS_w}\right]^{-1},\quad x=0,\ t>0 \quad (3)$$

$$S_w=S_w|_{P_c=0},\quad x=L,\ t>0 \quad (4)$$

In Eq. 1, the zero capillary pressure and zero gravity fractional water function is given by

$$F_w=\left[1+\frac{k_{ro}\mu_w}{k_{rw}\mu_o}\right]^{-1} \quad (5)$$

In Eqs. 1–5, $x$ and $t$ denote the distance and time, $\phi$ and $K$ are the porosity and permeability of the core plug, $u$ is the total volumetric flux of the pore fluids which is equal to the injection fluid volumetric flux under the considered conditions, $L$ is the length of the core plug, $\mu_o$ and $k_{ro}$ denote the viscosity and relative permeability of the oil, $\mu_w$ and $k_{rw}$ are the viscosity and relative permeability of the water, $P_c$ denotes the capillary pressure, $S_w$ is the water phase saturation in the core, $S_w^*(x)$ represents the initial water saturation distribution in the core, and $S_w|_{P_c=0}$ defines the water saturation corresponding to zero capillary pressure. For the problem considered by Kulkarni et al. (1998), $S_w^*(x)=1$ and $S_w|_{P_c=0}=1-S_{or}$, where the residual oil saturation is taken zero, $S_{or}=0$. $(u_o)_{inj}$ is the volumetric flux of the oil injected into the core plug.

Figure 1 shows a comparison of the fully-implicit numerical solutions of Eqs. 1–5, using a first-order accurate temporal and second-order accurate spatial finite difference approximations, with the solutions given by Kulkarni et al. (1998) in their Figure 6. As can be seen, the present solutions and those given by Kulkarni et al. (1998) are very close. The small differences are due to the possible differences in the numerical solution methods (time increments, grid point spacing, and discretization methods) and the type and number of differential equations. The numerical solution of the present model is faster and can be carried out more accurately, compared to the Kulkarni et al. (1998) model, because of the reduction in the number of equations. In view of the above discussion, it appears that the present model is advantageous for repetitive applications in simultaneous determination of the relative permeability and capillary pressure from routine laboratory immiscible displacement tests in core samples.

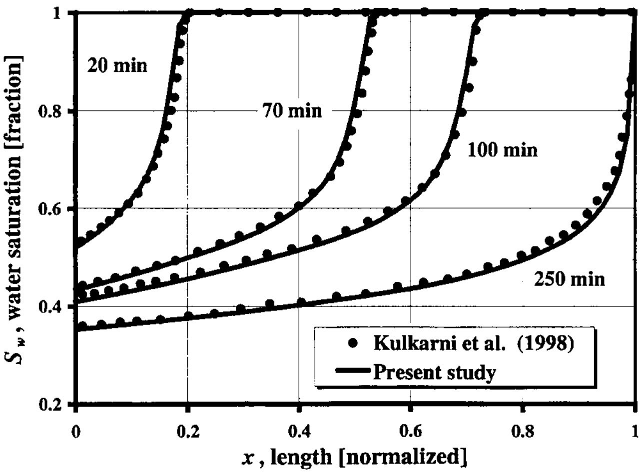


**Figure 1. Comparison of the present solutions with Kulkarni et al. (1998).**

### Literature cited

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## Reply:

Penuela and Civan suggest a method for solving the equations for two-phase flow in permeable media that is different from the method used by Kulkarni et al. (1998). They claim their method is more efficient computationally. In Kulkarni et al. (1998), the specific details associated with solving the model equations were not addressed, and none of the results depend on the relative computational efficiency for solving those equations.

There are several different ways to formulate and solve the equations used to simulate the core-flood experiments (Aziz and Settari, 1979). We use a fully implicit pressure-saturation formulation. This formulation is excellent as a general purpose core-flood simulator, since it can be used regardless of whether capillary effects are present or not, and regardless of whether the fluids are compressible or not. Penuela and Civan suggest a formulation which is tractable only for the incompressible case (Aziz and Settari, 1979). While they claim that the formulation leads to a single partial differential equation, that equation provides for only the saturation distribution, and not the pressure. A second differential equation must be solved for the pressure, which they did not consider. When both fluid states—the pressure and saturation—are determined, the decoupling of the saturation equation in their method may allow for a somewhat more efficient solution, although the computational reduction will not be so great as implied. Claims of greater accuracy are not supported. Differences in the saturation solutions shown in their figure probably result from the use of slightly different inputs for the properties, and do not necessarily reflect differences in the solution methods.


### Literature cited

Aziz, K., and A. Settari, *Petroleum Reservoir Simulation*, Applied Science Publishers, London (1979).

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